

q-1 Exploration #2

$$\textcircled{1} \frac{1}{(1+x^2)} = \frac{1}{1-(-x^2)} \\ = 1 - x^2 + x^4 - x^6 + x^8 - \dots + (-1)^n x^{2n} + \dots$$

$$\textcircled{2} \int_0^x \frac{1}{1+t^2} dt = \int_0^x (1 - t^2 + t^4 - t^6 + \dots + (-1)^n t^{2n}) dt$$

$$\tan^{-1} x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots - (-1)^n \frac{x^{2n+1}}{2n+1} + \dots$$

③ Yes

$$\textcircled{4} 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} ; \text{Yes}$$

Exploration #3

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

$$\textcircled{1} f'(x) = 1 + x + \frac{x^2}{2} + \dots \quad \text{same function}$$

$$\textcircled{2} f(0) = 1$$

$$\textcircled{3} f(x) = e^x ; \text{same derivative } f(0) = 1$$

$$\textcircled{4} y = f(x), \frac{dy}{dx} = y \quad \text{and when } y=1, x=0$$

$$\textcircled{5} \frac{1}{y} dy = dx$$

$$\ln y = x + C$$

$$y = Ce^x \quad y=1 \text{ when } x=0$$

$$1 = C$$

$$\underline{y = e^x}$$